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Author(s)	Michele, S.; Renzi, E.; Perez-Collazo, C.; Greaves, D.; Iglesias, Gregorio
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Power extraction in regular and random waves from an OWC in hybrid wind-wave energy systems

S.Michele^{a,*}, E.Renzi^a, C.Perez-Collazo^{b,c}, D.Greaves^c, G.Iglesias^{d,c}

^a*Department of Mathematical Sciences, Loughborough University, Leics LE11 3TU, UK*

^b*School of Mines and Energy Engineering, University of Vigo, Vigo, Pontevedra, Spain*

^c*School of Engineering, Computing and Mathematics, University of Plymouth, Drake Circus, Plymouth PL4 8AA, UK*

^d*Environmental Research Institute & School of Engineering, University College Cork, Western Rd., Cork, Ireland*

Abstract

A mathematical model is developed to analyse the hydrodynamics of a novel oscillating water column (OWC) in a hybrid wind-wave energy system. The OWC has a coaxial cylindrical structure in which the internal cylinder represents the mono-pile of an offshore wind turbine while the external cylinder has a skirt whose scope is to guide the wave energy flux inside the chamber. This layout is not casual, but consistent with the current approach to harnessing wave energy through hybrid systems. The device shape is rather complex and the boundary value problem is solved by applying the matching-method of eigenfunctions. Within the framework of a linearised theory, we model the turbine damping effects by assuming the airflow to be proportional to the air chamber pressure. Consequently, the velocity potential can be decomposed into radiation and diffraction problems. We study the effects of both skirt and internal radius dimensions on the power extraction efficiency for monochromatic and random waves. We show that the skirt has strong effects on the global behaviour, while the internal cylinder affects the values of the sloshing eigenfrequencies. Finally, we validate the analytical model with laboratory data and show a good agreement between analytical and

*Corresponding author

Email addresses: s.michele@lboro.ac.uk (S.Michele), e.renzi@lboro.ac.uk (E.Renzi), carlos.perez.collazo@uvigo.es (C.Perez-Collazo), deborah.greaves@plymouth.ac.uk (D.Greaves), gregorio.iglesias@ucc.ie (G.Iglesias)

experimental results.

Keywords: Wave energy, Wave-structure interactions, Oscillating water column

1. Introduction

The oscillating water column (OWC) is one of the most studied devices to extract energy from water waves. For an extensive review concerning related theories and experiments we refer to the works of McCormick (1981), Falnes (2002), Babarit et al. (2012) and Babarit (2018). Substantially, the OWC is a partially-immersed structure open at its bottom that confines air above the internal fluid free-surface. Incident waves induce oscillations inside the chamber, thus the airflow is forced to pass through a turbine usually located at the top of the OWC. The turbine is coupled to a generator to produce electricity. In this paper we perform a novel analysis of a coaxial cylindrical OWC in a hybrid wind-wave energy system.

Analytical theories concerning immersed floating structures open at one end have been applied in several contexts. One of the main contributions is due to Garrett (1970), who examined cylindrical bottomless harbours. Other authors such as Mavrakos (1985) analysed the effects of the wall thickness of a floating cylindrical body on the diffracted wave field. Extension of hydrodynamical theories to OWC devices includes the analysis of both bi-dimensional (Evans, 1982; Sarmiento and Falcão, 1985) and three-dimensional configurations. For example, Evans and Porter (1997) were the first to solve the case of a partially immersed cylindrical OWC in open sea by applying the Galerkin method to integral equations.

Several recent studies consider the OWC to be installed in fixed structures for coastal protection. This is mainly due to economical reasons and difficulties in developing wave energy absorbers on their own. Concerning analytical models for OWCs combined with external structures, Martin-Rivas and Mei (2009a) and Martin-Rivas and Mei (2009b) solved the linearised problem of an OWC at the tip of a breakwater and the case of an OWC installed on a straight coast. More recently, Lovas et al. (2010) extended the theory of Martin-Rivas and Mei (2009b) to examine the hydrodynamic wave field when the OWC is installed in correspondence of convex or concave corners, Deng et al. (2013) and Deng et al. (2014) took into account the presence of a coaxial supporting structure to examine possible benefits on the energy

33 conversion efficiency, while Zhou et al. (2018) solved the case of a concentric
34 axisymmetric OWC including an internal mono-pile. Wave-structure inter-
35 action in hybrid wave farms with OWC devices was investigated by Zheng
36 et al. (2018), while recently, breakwater-integrated OWCs were studied by
37 means of a semi-analytical model by Zheng et al. (2019).

38 Despite the large number of theories developed so far, the wave energy
39 sector is suffering from slow technological progress combined with difficulties
40 in attracting funds (Magagna and Uihlein, 2015). This is mainly due to
41 the large levelised cost of energy (LCOE) (Astariz et al., 2015, 2016). One
42 way to attract funds and confidence in industry is to combine both wind
43 and wave energy technologies. This is a recent research effort that aims to
44 develop a more sustainable and affordable device to extract clean energy
45 against fossil fuels. We refer to the work of Perez-Collazo et al. (2015) for an
46 extensive review of alternatives that combine wave and offshore wind energy
47 technologies.

48 Motivated by this recent technology concept, Perez-Collazo et al. (2018a)
49 and Perez-Collazo et al. (2018b) tested a 1:50 novel hybrid wind-wave system
50 that integrates a skirt, a cylindrical OWC and an offshore wind turbine on
51 a jacket frame structure. The Authors investigated experimentally the hy-
52 drodynamic response of the device to monochromatic and random waves for
53 different orifice diameters simulating different air turbines. Promising results
54 were obtained; however further analytical work is needed to understand the
55 influence of the device parameters on the global hydrodynamic behaviour of
56 the system.

57 For all these reasons, in this paper we develop a mathematical model for
58 the OWC designed by Perez-Collazo et al. (2018b) by adding a concentric
59 cylinder that represents a wind turbine installed on a mono-pile. This system
60 appears similar to the theoretical models already solved by the Authors pre-
61 viously mentioned. However, the skirt connected to the OWC removes any
62 axial symmetry and the boundary value problem increases in complexity.

63 Here we apply an eigenfunction expansion method and solve the corre-
64 sponding velocity potentials in terms of Bessel functions and modified Bessel
65 functions of the first and second kind, respectively (Linton and McIver, 2001;
66 Mei et al., 2005). First, we analyse the case of monochromatic waves and
67 derive several integral relations based on the Green's theorem which can be
68 useful to check the numerical computations of the radiation and diffraction
69 velocity potentials. Then we show that both the skirt and the internal cylin-
70 der play an important role in the power extraction and the sloshing dynamics

inside the air chamber.

Next, we extend the theory to the case of random waves described by the JONSWAP spectrum (Goda, 2000) and characterise the power extraction efficiency by applying the superposition principle to the different incident wave frequency components. We find that the resonant peaks related to the Helmholtz and sloshing modes decrease in intensity with respect to the monochromatic case, and that the random waves have a broadening effect on the capture factor curve. Interestingly, similar results were also found in the context of oscillating wave surge converters (Michele et al., 2016a,b; Sarkar et al., 2014).

Laboratory experimental models usually simulate the damping effects of a turbine by means of an orifice of a certain diameter connecting the air chamber with the atmosphere (Perez-Collazo et al., 2018b). In this case, the air turbine is of the impulse type; therefore, a quadratic relation exists between the airflow through the orifice and the pressure head between the air chamber and the atmosphere, thus the boundary condition on the free surface becomes nonlinear (Pereiras et al., 2015; López et al., 2016, 2014). We then non-dimensionalise the corresponding equation by using adequate scales (Michele et al., 2018, 2019a; Michele and Renzi, 2019; Sammarco et al., 1997a,b) and apply a perturbation technique to the velocity potential. We show that if the ratio between the orifice and OWC diameter is not very small, the air pressure inside the chamber and the corresponding airflow through the orifice are governed by the diffraction potential at the leading order. Finally, we compare our analytical model with the 1:50 scale model in Perez-Collazo et al. (2018b) and show good matching of the theoretical results with those evaluated experimentally.

2. Governing Equations

With reference to figure 1, consider an OWC device embedded in a hybrid wave-wind energy extraction system. Let us define a Cartesian reference system with the x and y -axes coincident with the undisturbed free-surface level and the z -axis pointing vertically upward. The concentric cylindrical structure of the OWC has inner radius equal to R_i , while the external radius corresponds to R_e . The internal cylinder spans the entire water depth and is fixed with the horizontal bottom at $z = -h$. The external cylinder does not have constant draught in water but includes a skirt of height h_s and corresponding arc-length $R_e(\theta_2 - \theta_1)$ with $\theta_2 > \theta_1$. The remaining part of

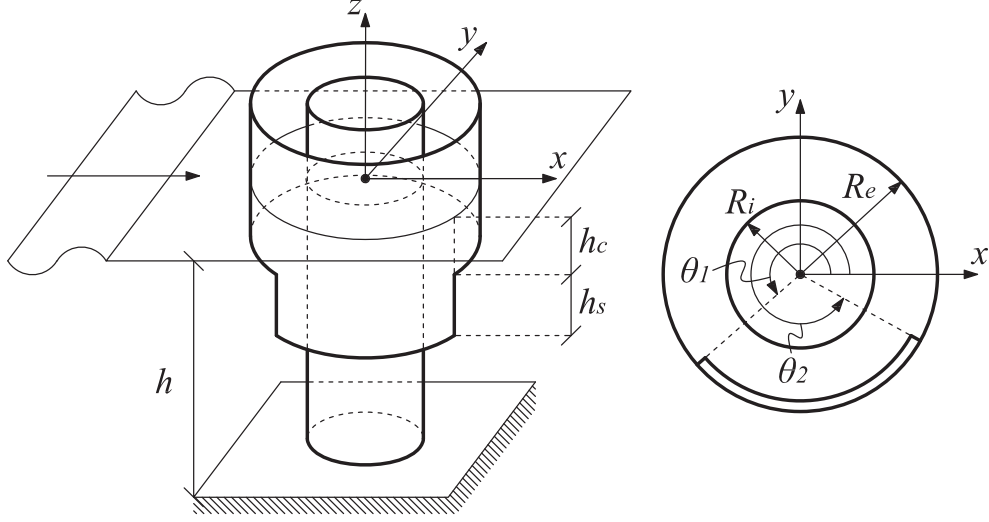


Figure 1: Three-dimensional representation and horizontal cross-section of the hybrid wind-wave energy converter.

the external structure has draught equal to h_c . The chamber is open at its base, while at the top an energy conversion system transforms the airflow through it in electricity (Falnes, 2002). Let us define the solid wetted surfaces of the OWC

$$S_{R_i} = \{r = R_i, \theta \in [0, 2\pi), z \in [-h, 0]\}, \quad (1)$$

$$S_{R_e} = \{r = R_e, \theta \in [0, 2\pi), z \in [-h_c, 0]\} \cup \{r = R_e, \theta \in [\theta_1, \theta_2], z \in [-h_c - h_s, -h_c]\}, \quad (2)$$

and the fluid surface S_f representing the gap under the OWC,

$$S_f = \{r = R_e, \theta \in [0, 2\pi), z \in [-h, 0]\} \setminus S_{R_e}, \quad (3)$$

where $r = \sqrt{x^2 + y^2}$ represents the radial coordinate and θ is the angular coordinate positive anticlockwise. Moreover, let us define the following surfaces

$$S_i = \{r \in [R_i, R_e], \theta \in [0, 2\pi), z = 0\}, \quad (4)$$

$$S_e = \{r \in [R_e, \infty), \theta \in [0, 2\pi), z = 0\}, \quad (5)$$

where S_i denotes the free surface inside the chamber, while S_e is the external free surface in contact with air at constant atmospheric pressure. The fluid

118 is inviscid and incompressible, while the fluid flow can be assumed to be irro-
 119 tational. Then the governing equation for the velocity potential $\Phi(x, y, z, t)$
 120 satisfies the Laplace equation in the fluid domain $\Omega(x, y, z)$. On the free
 121 surfaces S_e and S_i we have the linearised kinematic condition

$$\zeta_t = \Phi_z, \quad z = 0, \quad (6)$$

122 and the linearised mixed boundary conditions (Mei et al., 2005)

$$\Phi_{tt} + g\Phi_z = 0, \quad \text{on } S_e, \quad (7)$$

$$\Phi_{tt} + g\Phi_z = -\frac{P_{at}}{\rho}, \quad \text{on } S_i, \quad (8)$$

123 where ζ represents the free-surface elevation, g is the acceleration due to
 124 gravity, P_a denotes the oscillating pressure of the air inside the chamber
 125 depending on time t and ρ is the water density. We further require tangential
 126 fluid velocity at the bottom and on the solid surfaces, hence

$$\Phi_n = 0, \quad \text{on solid boundaries}, \quad (9)$$

127 where n denotes the normal derivative to the relevant surface. The problem is
 128 forced by monochromatic incident waves of frequency ω , hence let us assume
 129 harmonic motion

$$\{\Phi, \zeta, P_a\} = \text{Re} \{(\phi, \eta, p_a) e^{-i\omega t}\}, \quad (10)$$

130 with i being the imaginary unit. We shall now write the Laplace equation
 131 and the boundary conditions (6)-(9) in terms of the spatial variables (ϕ, η, p_a)
 132 only. Thus we get

$$\nabla^2 \phi = 0, \quad \text{in } \Omega, \quad (11)$$

$$-i\omega\eta = \phi_z, \quad z = 0, \quad (12)$$

$$-\omega^2 \phi + g\phi_z = 0, \quad \text{on } S_e, \quad (13)$$

$$-\omega^2 \phi + g\phi_z = i\omega \frac{p_a}{\rho}, \quad \text{on } S_i, \quad (14)$$

$$\phi_n = 0, \quad \text{on solid boundaries}. \quad (15)$$

133 Finally we require that the velocity potential ϕ be outgoing for $r \rightarrow \infty$.

134 Following the method of Evans and Porter (1997) we decompose the ve-
 135 locity potential in two parts, i.e.

$$\phi = \phi^D + \phi^R, \quad (16)$$

136 where ϕ^D is the diffraction potential satisfying the boundary conditions (12)-
 137 (15) with $p_a = 0$ and ϕ^R is the radiation potential that satisfies the same
 138 conditions with the unknown forcing pressure $p_a \neq 0$. Let us decompose
 139 the fluid domain Ω by defining Ω_i and Ω_e , respectively, as the internal and
 140 external fluid subdomains:

$$\Omega_i = \{r \in (R_i, R_e), \theta \in [0, 2\pi), z \in (-h, 0)\}, \quad (17)$$

$$\Omega_e = \{r \in (R_e, \infty), \theta \in [0, 2\pi), z \in (-h, 0)\}, \quad (18)$$

142 and let ϕ_i^D (ϕ_i^R) be the diffraction (radiation) potential in $(r, \theta, z) \in \Omega_i$ and
 143 ϕ_e^D (ϕ_e^R) the diffraction (radiation) potentials in $(r, \theta, z) \in \Omega_e$.

144 The boundary value problem for the external velocity potentials $\phi_e^{D,R}$ is

$$\nabla^2 \phi_e^{D,R} = 0, \quad \text{in } \Omega_e, \quad (19)$$

$$-\omega^2 \phi_e^{D,R} + g \phi_{e_z}^{D,R} = 0, \quad \text{on } S_e, \quad (20)$$

$$\phi_{e_r}^{D,R} = 0, \quad \text{on } S_{R_e}, \quad (21)$$

$$\phi_{e_z}^{D,R} = 0, \quad z = -h \quad (22)$$

$$\phi_e^{D,R} = \phi_i^{D,R}, \quad \text{on } S_f, \quad (23)$$

$$\phi_{e_r}^{D,R} = \phi_{i_r}^{D,R}, \quad r = R_e, \quad (24)$$

145 where the conditions (23)-(24) represent respectively continuity of the po-
 146 tential (pressure) and of the velocity field between the external velocity po-
 147 tentials $\phi_e^{D,R}$ and the internal velocity potentials $\phi_i^{D,R}$. The boundary value
 148 problem for the internal velocity potentials $\phi_i^{D,R}$ is governed by

$$\nabla^2 \phi_i^{D,R} = 0, \quad \text{in } \Omega_i, \quad (25)$$

$$-\omega^2 \phi_i^D + g \phi_{i_z}^D = 0, \quad \text{on } S_i, \quad (26)$$

$$-\omega^2 \phi_i^R + g \phi_{i_z}^R = i\omega \frac{p_a}{\rho}, \quad \text{on } S_i, \quad (27)$$

$$\phi_{i_r}^{D,R} = 0, \quad \text{on } S_{R_i}, \quad (28)$$

$$\phi_{i_r}^{D,R} = 0, \quad \text{on } S_{R_e}, \quad (29)$$

$$\phi_{i_z}^{D,R} = 0, \quad z = -h, \quad (30)$$

149 and the coupling matching conditions (23)-(24).

150 In the following sections we solve the diffraction and radiation potential in
 151 Ω_e and Ω_i by integrating the matching conditions on the common boundaries.

152 2.1. Diffraction potential solution

153 Let us assume for simplicity incident waves with direction parallel to the
 154 x -axis and amplitude A . The generalized angles θ_1 and θ_2 can be properly
 155 modified in order to investigate the effects of oblique incident waves. For the
 156 sake of example, a skirt described by the angles $\bar{\theta}_1 = \theta_1 + \alpha$ and $\bar{\theta}_2 = \theta_2 + \alpha$
 157 simulates the effects of incoming waves with angle of incidence $\pi - \alpha$ on the
 158 same OWC.

159 Use of cylindrical coordinates yields the following general solution for the
 160 diffraction potential in $(r, \theta, z) \in \Omega_e$

$$\begin{aligned} \phi_e^D = & -\frac{iAg}{\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} \left\{ \mathcal{B}_{n0}^D \frac{\sin n\theta H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \right|_{r=R_e} \right. \\ & \left. + \cos n\theta \left[\epsilon_n i^n J_n(k_0 r) + \mathcal{A}_{n0}^D \frac{H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \right]_{r=R_e} \right\} + \\ & \sum_{l=1}^{\infty} \frac{(\mathcal{A}_{nl}^D \cos n\theta + \mathcal{B}_{nl}^D \sin n\theta) K_n(\bar{k}_l r) \cosh k_l (h+z)}{K_{n_r}(\bar{k}_l r)|_{r=R_e} \cosh k_l h} \Bigg\}, \end{aligned} \quad (31)$$

161 while the diffraction potential in $(r, \theta, z) \in \Omega_i$ can be written as

$$\begin{aligned} \phi_i^D = & -\frac{iAg}{\omega} \\ & \times \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} \left\{ \cos n\theta \left[\mathcal{C}_{n0}^D \frac{J_n(k_0 r)}{J_{n_r}(k_0 r)} \right|_{r=R_i} + \mathcal{D}_{n0}^D \frac{Y_n(k_0 r)}{Y_{n_r}(k_0 r)} \right|_{r=R_i} \right] \right. \\ & \left. + \sin n\theta \left[\mathcal{E}_{n0}^D \frac{J_n(k_0 r)}{J_{n_r}(k_0 r)} \right|_{r=R_i} + \mathcal{F}_{n0}^D \frac{Y_n(k_0 r)}{Y_{n_r}(k_0 r)} \right|_{r=R_i} \right] \right\} \\ & + \sum_{l=1}^{\infty} \frac{\cosh k_l (h+z)}{\cosh k_l h} \left\{ \cos n\theta \left[\mathcal{C}_{nl}^D \frac{I_n(\bar{k}_l r)}{I_{n_r}(\bar{k}_l r)} \right|_{r=R_i} + \mathcal{D}_{nl}^D \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)} \right|_{r=R_i} \right] \right. \\ & \left. + \sin n\theta \left[\mathcal{E}_{nl}^D \frac{I_n(\bar{k}_l r)}{I_{n_r}(\bar{k}_l r)} \right|_{r=R_i} + \mathcal{F}_{nl}^D \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)} \right|_{r=R_i} \right] \right\}. \end{aligned} \quad (32)$$

162 In the latter expressions, A denotes the amplitude of the incident waves, ϵ_n
 163 is the Jacobi symbol defined as

$$\epsilon_0 = 1, \quad \epsilon_n = 2 \quad n = 1, \dots, \infty, \quad (33)$$

164 the terms k_l 's are the roots of the dispersion relation (Mei et al., 2005)

$$\left. \begin{aligned} \omega^2 &= gk_0 \tanh k_0 h, \\ \omega^2 &= -g\bar{k}_l \tan \bar{k}_l h, \quad k_l = i\bar{k}_l, \quad l = 1, \dots, \infty \end{aligned} \right\}, \quad (34)$$

165 J_n and Y_n are the Bessel functions of order n , $H_n^{(1)}$ is the Hankel function
 166 of the first kind and order n , I_n and K_n are the modified Bessel functions
 167 of order n and finally $\mathcal{A}_{nl}^D, \mathcal{B}_{nl}^D, \mathcal{C}_{nl}^D, \mathcal{D}_{nl}^D, \mathcal{E}_{nl}^D, \mathcal{F}_{nl}^D$ are complex constants yet
 168 unknown. The no-flux condition (28) yields

$$\mathcal{C}_{nl}^D = -\mathcal{D}_{nl}^D, \quad \mathcal{E}_{nl}^D = -\mathcal{F}_{nl}^D, \quad (35)$$

169 thus expression (32) reads now

$$\begin{aligned} \phi_i^D &= -\frac{iAg}{\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} (\cos n\theta \mathcal{D}_{n0}^D + \sin n\theta \mathcal{F}_{n0}^D) \mathcal{T}_n \right. \\ &\quad \left. + \sum_{l=1}^{\infty} \frac{\cosh k_l (h+z)}{\cosh k_l h} (\cos n\theta \mathcal{D}_{nl}^D + \sin n\theta \mathcal{F}_{nl}^D) \mathcal{U}_{nl} \right\}, \end{aligned} \quad (36)$$

170 where

$$\mathcal{T}_n = \frac{Y_n(k_0 r)}{Y_{n_r}(k_0 r)|_{r=R_i}} - \frac{J_n^{(1)}(k_0 r)}{J_{n_r}(k_0 r)|_{r=R_i}}, \quad \mathcal{U}_{nl} = \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)|_{r=R_i}} - \frac{I_n(\bar{k}_l r)}{I_{n_r}(\bar{k}_l r)|_{r=R_i}}. \quad (37)$$

171 Substituting expressions (36) and (31) in the matching condition (24) and
 172 integrating over $S_f \cup S_{R_e}$, gives

$$\epsilon_n i^n J_{n_r}(k_0 r)|_{r=R_e} + \mathcal{A}_{n0}^D = \mathcal{D}_{n0}^D \mathcal{T}_{n_r}|_{r=R_e}, \quad \mathcal{B}_{n0}^D = \mathcal{F}_{n0}^D \mathcal{T}_{n_r}|_{r=R_e}, \quad (38)$$

173

$$\mathcal{A}_{nl}^D = \mathcal{D}_{nl}^D \mathcal{U}_{nl_r}|_{r=R_e}, \quad \mathcal{B}_{nl}^D = \mathcal{F}_{nl}^D \mathcal{U}_{nl_r}|_{r=R_e}. \quad (39)$$

174 The external diffraction potential can be written in terms of the coefficients
 175 \mathcal{D}^D and \mathcal{F}^D

$$\begin{aligned} \phi_e^D = & -\frac{iAg}{\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} \left\{ \cos n\theta \left[\epsilon_n i^n J_n(k_0 r) + \frac{H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \right]_{r=R_e} \right. \right. \\ & \times \left. \left. \left(\mathcal{D}_{n0}^D \mathcal{T}_{n_r}|_{r=R_e} - \epsilon_n i^n J_{n_r}(k_0 r)|_{r=R_e} \right) \right] + \mathcal{F}_{n0}^D \mathcal{T}_{n_r}|_{r=R_e} \frac{\sin n\theta H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \right\}_{r=R_e} \\ & + \sum_{l=1}^{\infty} \frac{\cosh k_l (h+z)}{\cosh k_l h} \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)|_{r=R_e}} \mathcal{U}_{nl_r}|_{r=R_e} \left(\mathcal{D}_{nl}^D \cos n\theta + \mathcal{F}_{nl}^D \sin n\theta \right) \Bigg\}. \end{aligned} \quad (40)$$

176 For the sake of brevity, we introduce the following integrals for the vertical
 177 eigenfunctions

$$I_{sl}^{(1)} = \int_{-h}^{-hc-hs} \frac{\cosh k_l (h+z) \cosh k_s (h+z)}{\cosh k_l h \cosh k_s h} dz, \quad (41)$$

$$I_{sl}^{(2)} = \int_{-hc-hs}^{-hc} \frac{\cosh k_l (h+z) \cosh k_s (h+z)}{\cosh k_l h \cosh k_s h} dz, \quad (42)$$

$$I_{sl}^{(3)} = \int_{-hc}^0 \frac{\cosh k_l (h+z) \cosh k_s (h+z)}{\cosh k_l h \cosh k_s h} dz, \quad (43)$$

180 and the following integrals involving the angular eigenfunctions

$$cc_{pn} = \int_{\theta_2}^{\theta_1} \cos n\theta \cos p\theta d\theta, \quad ss_{pn} = \int_{\theta_2}^{\theta_1} \sin n\theta \sin p\theta d\theta, \quad (44)$$

$$sc_{pn} = \int_{\theta_2}^{\theta_1} \sin n\theta \cos p\theta d\theta, \quad cs_{pn} = \int_{\theta_2}^{\theta_1} \cos n\theta \sin p\theta d\theta, \quad (45)$$

182 whose values can be found straightforwardly.

183 Multiplying the condition (23) by $\cosh k_s (h + z) \cos p\theta / \cosh k_s h$ and in-
 184 tegrating over the fluid surface S_f , yields

$$\begin{aligned}
 & \frac{4I_{s0}^{(1)} \epsilon_n i^{n+1} \delta_{pn}}{\epsilon_p R_e H_{n_r}^{(1)}(k_0 r) \Big|_{r=R_e}} + \sum_{m=0}^{\infty} \frac{2cc_{pm} I_{s0}^{(2)} \epsilon_m i^{m+1}}{R_e \pi H_{m_r}^{(1)}(k_0 r) \Big|_{r=R_e}} = \frac{2\mathcal{D}_{n0}^D I_{s0}^{(1)} \delta_{pn} \pi \tau_n}{\epsilon_p} \\
 & + \sum_{m=0}^{\infty} I_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^D cc_{mn} + \mathcal{F}_{m0}^D sc_{pm}) + \sum_{l=1}^{\infty} \frac{2\mathcal{D}_{nl}^D I_{sl}^{(1)} \delta_{pn} \pi \gamma_{nl}}{\epsilon_p} \\
 & + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} I_{s0}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^D cc_{pm} + \mathcal{F}_{ml}^D sc_{pm}), \quad s = 0, 1, \dots, \quad p = 0, 1, \dots, \quad (46)
 \end{aligned}$$

185 where δ_{pn} is the Kronecker delta, while τ_m and γ_{ml} have the following expres-
 186 sions

$$\tau_m = \mathcal{T}_m \Big|_{r=R_e} - \frac{H_m^{(1)}(k_0 r) \Big|_{r=R_e}}{H_{m_r}^{(1)}(k_0 r) \Big|_{r=R_e}} \mathcal{T}_{m_r} \Big|_{r=R_e}, \quad (47)$$

187

$$\gamma_{ml} = \mathcal{U}_{ml} \Big|_{r=R_e} - \frac{K_m(\bar{k}_l r) \Big|_{r=R_e}}{K_{m_r}(\bar{k}_l r) \Big|_{r=R_e}} \mathcal{U}_{ml_r} \Big|_{r=R_e}. \quad (48)$$

188 Multiplying again the condition (23) by $\cosh k_s (h + z) \sin p\theta / \cosh k_s h$ and
 189 integrating over the fluid surface S_f gives now

$$\begin{aligned}
 & \pi \mathcal{F}_{n0}^D I_{s0}^{(1)} \delta_{pn} \tau_n + \sum_{m=0}^{\infty} I_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^D cs_{pm} + \mathcal{F}_{m0}^D ss_{pn}) \\
 & + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^D I_{sl}^{(1)} \delta_{pn} \gamma_{nl} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} I_{sl}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^D cs_{pm} + \mathcal{F}_{ml}^D ss_{pm}) = 0. \quad (49)
 \end{aligned}$$

190 An additional condition for the internal diffraction potential can be obtained
 191 by multiplying $\phi_{i,r}^D = 0$ respectively by $\cosh k_s (h + z) \cos p\theta / \cosh k_s h$ and
 192 $\cosh k_s (h + z) \sin p\theta / \cosh k_s h$. Integrating over the relevant domain S_{R_e}
 193 gives us

$$\begin{aligned}
 & \sum_{l=1}^{\infty} \frac{2\pi \mathcal{D}_{nl}^D I_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nl_r} \Big|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} I_{sl}^{(2)} \mathcal{U}_{ml_r} \Big|_{r=R_e} (\mathcal{D}_{ml}^D \overline{cc}_{pm} + \mathcal{F}_{ml}^D \overline{sc}_{pm}) \\
 & + \frac{2\pi \delta_{pn} I_{s0}^{(3)} \mathcal{D}_{n0}^D \mathcal{T}_{n_r} \Big|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} I_{s0}^{(2)} \mathcal{T}_{m_r} \Big|_{r=R_e} (\mathcal{D}_{m0}^D \overline{cc}_{pm} + \mathcal{F}_{m0}^D \overline{sc}_{pm}) = 0, \quad (50)
 \end{aligned}$$

$$\begin{aligned}
& \pi \delta_{pn} I_{s0}^{(3)} \mathcal{F}_{n0}^D \mathcal{T}_{n_r}|_{r=R_e} + \sum_{m=0}^{\infty} I_{s0}^{(2)} \mathcal{T}_{m_r}|_{r=R_e} (\mathcal{D}_{m0}^D \overline{\mathcal{C}}_{s_{pm}} + \mathcal{F}_{m0}^D \overline{\mathcal{S}}_{s_{pm}}) \\
& + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^D I_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nl_r}|_{r=R_e} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} I_{sl}^{(2)} \mathcal{U}_{ml_r}|_{r=R_e} (\mathcal{D}_{ml}^D \overline{\mathcal{C}}_{s_{pm}} + \mathcal{F}_{ml}^D \overline{\mathcal{S}}_{s_{pm}}) = 0,
\end{aligned} \tag{51}$$

194 where $\overline{\mathcal{C}}\mathcal{C}, \overline{\mathcal{S}}\mathcal{S}, \overline{\mathcal{C}}\mathcal{S}, \overline{\mathcal{S}}\mathcal{C}$ are defined as follows

$$\overline{\mathcal{C}}\mathcal{C}_{pn} = \int_{\theta_1}^{\theta_2} \cos n\theta \cos p\theta \, d\theta, \quad \overline{\mathcal{S}}\mathcal{S}_{pn} = \int_{\theta_1}^{\theta_2} \sin n\theta \sin p\theta \, d\theta, \tag{52}$$

195

$$\overline{\mathcal{S}}\mathcal{C}_{pn} = \int_{\theta_1}^{\theta_2} \sin n\theta \cos p\theta \, d\theta, \quad \overline{\mathcal{C}}\mathcal{S}_{pn} = \int_{\theta_1}^{\theta_2} \cos n\theta \sin p\theta \, d\theta. \tag{53}$$

196 Summation of (46) and (50) plus summation of (49) and (51) gives an
 197 inhomogeneous linear system in the unknown coefficients \mathcal{D}^D and \mathcal{F}^D . Once
 198 they are known, the coefficients \mathcal{A}^D and \mathcal{B}^D can be obtained through (38)-
 199 (39), while \mathcal{C}^D and \mathcal{E}^D are given by (35).

200 2.2. Radiation potential solution

201 The problem is linear, hence the radiation velocity potential outside and
 202 inside the OWC can be assumed proportional to the pressure acting on the
 203 free surface S_i . The general solutions are similar to (31)-(32):

$$\begin{aligned}
\phi_e^R = & -\frac{ip_a}{\rho\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0(h+z)}{\cosh k_0 h} \frac{H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)|_{r=R_e}} (\mathcal{A}_{n0}^R \cos n\theta + \mathcal{B}_{n0}^R \sin n\theta) \right. \\
& \left. + \sum_{l=1}^{\infty} \frac{\cosh k_l(h+z)}{\cosh k_l h} \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)|_{r=R_e}} (\mathcal{A}_{nl}^R \cos n\theta + \mathcal{B}_{nl}^R \sin n\theta) \right\}, \tag{54}
\end{aligned}$$

$$\begin{aligned}
\phi_i^R = & -\frac{ip_a}{\rho\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0(h+z)}{\cosh k_0 h} \mathcal{T}_n (\mathcal{D}_{n0}^R \cos n\theta + \mathcal{F}_{n0}^R \sin n\theta) \right. \\
& \left. + \sum_{l=1}^{\infty} \frac{\cosh k_l(h+z)}{\cosh k_l h} \mathcal{U}_{nl} (\mathcal{D}_{nl}^R \cos n\theta + \mathcal{F}_{nl}^R \sin n\theta) \right\} - \frac{ip_a}{\rho\omega}, \tag{55}
\end{aligned}$$

204 except for the forcing term that takes into account for the pressure p_a on S_i .
 205 Continuity of the fluid velocity across the cylindrical surface $r = R_e$ yields

$$\mathcal{A}_{n0}^R = \mathcal{D}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e}, \quad \mathcal{B}_{n0}^R = \mathcal{F}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e}, \quad (56)$$

206

$$\mathcal{A}_{nl}^R = \mathcal{D}_{nl}^R \mathcal{U}_{nl_r}|_{r=R_e}, \quad \mathcal{B}_{nl}^R = \mathcal{F}_{nl}^R \mathcal{U}_{nl_r}|_{r=R_e}, \quad (57)$$

207 while multiplying the condition (23) by $\cosh k_s (h + z) \cos p\theta / \cosh k_s h$ and
 208 integrating over S_f gives

$$\begin{aligned} & -2\pi\delta_{p0} \frac{\sinh k_s (h - h_c - h_s)}{k_s \cosh k_s h} - \text{cc}_{p0} \frac{\sinh k_s (h - h_c) - \sinh k_s (h - h_c - h_s)}{k_s \cosh k_s h} = \\ & \frac{2\mathcal{D}_{n0}^R \mathcal{I}_{s0}^{(1)} \delta_{pn} \pi \tau_n}{\epsilon_p} + \sum_{m=0}^{\infty} \mathcal{I}_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^R \text{cc}_{pm} + \mathcal{F}_{m0}^R \text{sc}_{pm}) + \sum_{l=1}^{\infty} \frac{2\mathcal{D}_{nl}^R \mathcal{I}_{sl}^{(1)} \delta_{pn} \pi \gamma_{nl}}{\epsilon_p} \\ & + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathcal{I}_{s0}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^R \text{cc}_{pm} + \mathcal{F}_{ml}^R \text{sc}_{pm}). \end{aligned} \quad (58)$$

209 Similarly, multiplying continuity condition (23) by $\cosh k_s (h + z) \sin p\theta / \cosh k_s h$
 210 and integrating over the surface S_f yields

$$\begin{aligned} & \pi \mathcal{F}_{n0}^R \mathcal{I}_{s0}^{(1)} \delta_{pn} \tau_n + \sum_{m=0}^{\infty} \mathcal{I}_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^R \text{cs}_{pm} + \mathcal{F}_{m0}^R \text{ss}_{pm}) \\ & + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathcal{I}_{sl}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^R \text{cs}_{pm} + \mathcal{F}_{ml}^R \text{ss}_{pm}) + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^R \mathcal{I}_{sl}^{(1)} \delta_{pn} \gamma_{nl} \\ & = \frac{(\cos p\theta_2 - \cos p\theta_1) [\sinh k_s (h - h_c) - \sinh k_s (h - h_c - h_s)]}{pk_s \cosh k_s h}. \end{aligned} \quad (59)$$

211 Finally the condition $\phi_{i_r}^R = 0$ on S_{Re} allows to obtain

$$\begin{aligned} & \frac{2\pi\delta_{pn} \mathcal{I}_{s0}^{(3)} \mathcal{D}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} \mathcal{I}_{s0}^{(2)} \mathcal{T}_{m_r}|_{r=R_e} (\mathcal{D}_{m0}^R \overline{\text{cc}}_{pm} + \mathcal{F}_{m0}^R \overline{\text{sc}}_{pm}) \\ & + \sum_{l=1}^{\infty} \frac{2\pi \mathcal{D}_{nl}^R \mathcal{I}_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nl_r}|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathcal{I}_{sl}^{(2)} \mathcal{U}_{ml_r}|_{r=R_e} (\mathcal{D}_{ml}^R \overline{\text{cc}}_{pm} + \mathcal{F}_{ml}^R \overline{\text{sc}}_{pm}) = 0, \end{aligned} \quad (60)$$

$$\begin{aligned}
& \pi \delta_{pn} I_{s0}^{(3)} \mathcal{F}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e} + \sum_{m=0}^{\infty} I_{s0}^{(2)} \mathcal{T}_{m_r}|_{r=R_e} (\mathcal{D}_{m0}^R \overline{\mathcal{CS}}_{pm} + \mathcal{F}_{m0}^R \overline{\mathcal{SS}}_{pm}) \\
& + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^R I_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nl_r}|_{r=R_e} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} I_{sl}^{(2)} \mathcal{U}_{ml_r}|_{r=R_e} (\mathcal{D}_{ml}^R \overline{\mathcal{CS}}_{pm} + \mathcal{F}_{ml}^R \overline{\mathcal{SS}}_{pm}) = 0.
\end{aligned} \tag{61}$$

As in the case of the diffraction potential, summation of equation (58) and (60) and summation of (59) and (61) yield an inhomogeneous linear system in \mathcal{D}^R and \mathcal{F}^R . The remaining constants for the external velocity potential $\mathcal{A}^R, \mathcal{B}^R$ can be evaluated by applying (56)-(57).

3. Power extraction efficiency in regular waves and integral relations

Within the framework of a linear theory, the relation between the flux rate Q through the turbine and the air pressure inside the chamber P_a can be defined as follows (Martin-Rivas and Mei, 2009b)

$$q = \left(\frac{KD}{N\rho_a} - \frac{i\omega V_0}{c_a^2 \rho_a} \right) p_a, \tag{62}$$

where K is an empirical coefficient depending on the turbine characteristics, D is the outer diameter of the turbine rotor, ρ_a the air density, V_0 the volume of air in the chamber when $\eta = 0$, c_a the speed of sound in air, while q represents the complex part of Q independent on time, i.e.

$$Q = \text{Re} \{ q e^{-i\omega t} \} = \text{Re} \left\{ e^{-i\omega t} \int_{S_i} \frac{\partial \Phi}{\partial z} dS_i \right\}. \tag{63}$$

The flux q can be further decomposed into radiation and diffraction components

$$q = q^D + q^R, \quad q^D = \int_{S_i} \frac{\partial \phi_i^D}{\partial z} dS_i = \Gamma A, \quad q^R = \int_{S_i} \frac{\partial \phi_i^R}{\partial z} dS_i = -(\mathcal{B} - i\mathcal{C}) p_a, \tag{64}$$

228 with Γ being the complex exciting force, while the real quantities \mathcal{B} and \mathcal{C}
 229 represent respectively the radiation damping and the added mass due to the
 230 radiation wave field. The averaged power output over a wave period is

$$\overline{P}_{out} = \frac{KD}{2N\rho_a} |p_a|^2 = \frac{A^2 |\Gamma|^2 KD}{2N\rho_a \left[\left(\frac{KD}{N\rho_a} + \mathcal{B} \right)^2 + \left(\frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2 \right]}, \quad (65)$$

231 hence the corresponding capture factor can be defined as the ratio between
 232 the generated power (65) and the energy influx of incident waves with am-
 233 plitude A per OWC width $2R_e$ (Michele et al., 2016b)

$$C_F = \frac{|\Gamma|^2 KD}{2\rho g R_e C_g N\rho_a \left[\left(\frac{KD}{N\rho_a} + \mathcal{B} \right)^2 + \left(\frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2 \right]}, \quad (66)$$

234 where C_g is the group velocity (Mei et al., 2005). Maximum efficiency of
 235 the capture factor (66) can be achieved if both the resonance condition, i.e.
 236 $(\mathcal{C} - \omega V_0/c_a^2 \rho_a) = 0$, and the identity $\mathcal{B} = KD/N\rho_a$ are satisfied. However,
 237 resonance is not always possible, mainly because of the difficulty in changing
 238 the structural parameter $V_0/c_a^2 \rho_a$. In any case, if $(\mathcal{C} - \omega V_0/c_a^2 \rho_a) \neq 0$, the
 239 optimal damping force exerted by the turbine can be chosen such that the
 240 derivative of C_F (66) with respect to $KD/N\rho_a$ is zero. This condition holds
 241 if

$$\frac{KD}{N\rho_a} = \sqrt{\mathcal{B}^2 + \left(\frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2}. \quad (67)$$

242 Substitution of the latter expression in (66) yields the optimized capture
 243 factor C_{Fopt}

$$C_{Fopt} = \frac{|\Gamma|^2 \sqrt{\mathcal{B}^2 + \left(\frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2}}{2\rho g R_e C_g \left[\left(\sqrt{\mathcal{B}^2 + \left(\frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2} + \mathcal{B} \right)^2 + \left(\frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2 \right]}. \quad (68)$$

244 If also resonance occurs, expression (68) yields the the maximum value of the
 245 capture factor C_{Fmax}

$$C_{Fmax} = \frac{|\Gamma|^2}{8\rho g R_e C_g \mathcal{B}}. \quad (69)$$

246 Note that in the case of axisymmetric bodies, i.e. without the skirt, the
 247 latter relation yields after some algebra

$$C_{Fmax} = \frac{1}{2k_0 R_e}, \quad (70)$$

248 i.e. two times smaller than the maximum that can be reached by an oscil-
 249 lating flap-type wave surge converter in open sea having width equal to $2R_e$
 250 (Michele et al., 2016b). Now we derive several integral relations to perform a
 251 numerical check of the hydrodynamic quantities Γ and \mathcal{B} . Applying Green's
 252 theorem to ϕ^R and its complex conjugate over the entire fluid domain Ω
 253 yields

$$\frac{1}{\rho\omega} \text{Re} \left\{ \int_{S_i} p^* \phi_i^R dS_i \right\} = \text{Re} \left\{ \int_{S_\infty} -i\phi_e^R \phi_e^{R*} dS_\infty \right\}, \quad (71)$$

254 where $(\cdot)^*$ denotes the complex conjugate of (\cdot) and S_∞ is a vertical cylinder
 255 of large radius $r \rightarrow \infty$ and height h . The radiation potential ϕ_e^R in the far
 256 field can be approximated by

$$\phi_e^R \sim \frac{\mathcal{A}(\theta)}{\sqrt{k_0 r}} e^{ik_0 r} \cosh k_0 (h + z), \quad (72)$$

257 in which $\mathcal{A}(\theta)$ represents the angular variation of the radiated waves at large
 258 distances. From (54) we get

$$\mathcal{A}(\theta) = -\frac{ip_a}{\rho\omega \cosh k_0 h} \sqrt{\frac{2}{\pi}} \sum_{n=0}^{\infty} \frac{e^{-i(\pi/4+n\pi/2)} (\mathcal{A}_{n0}^R \cos n\theta + \mathcal{B}_{n0}^R \sin n\theta)}{H_{n_r}^{(1)}(k_0 r) \Big|_{r=R_e}}. \quad (73)$$

259 Substitution of (64), (72) and (73) into the integrals (71) yields after some
 260 algebra

$$\begin{aligned} \mathcal{B} &= \frac{\rho\omega C_0}{|p_a|^2} \int_0^{2\pi} |\mathcal{A}(\theta)|^2 d\theta \\ &= \frac{2D_0}{\rho\omega \cosh^2 k_0 h} \left(\frac{2|\mathcal{A}_{00}^R|^2}{H_{0_r}^{(1)}(k_0 r) \Big|_{r=R_e}} + \sum_{n=1}^{\infty} \frac{|\mathcal{A}_{n0}^R|^2 + |\mathcal{B}_{n0}^R|^2}{H_{n_r}^{(1)}(k_0 r) \Big|_{r=R_e}} \right), \end{aligned} \quad (74)$$

261 where

$$C_0 = \int_{-h}^0 \cosh^2 k_0 (h + z) dz = \frac{2k_0 h + \sinh 2k_0 h}{4k_0}. \quad (75)$$

Expression (74) relates the radiation damping and the amplitude of the radiated waves at large distances and can be used for numerical check purposes. Similarly, applying Green's theorem to the radiation and diffraction velocity potentials gives

$$\frac{ip}{\rho\omega} \int_{S_i} \phi_{iz}^D dS_i = - \int_{S_\infty} (\phi^I \phi_{er}^R - \phi_e \phi_r^I) dS_\infty, \quad (76)$$

where ϕ^I is the velocity potential of the incident waves with amplitude A and frequency ω directed along the x -axis

$$\phi^I = -\frac{iAg \cosh k_0(h+z)}{\omega \cosh k_0 h} e^{ik_0 r \cos \theta}. \quad (77)$$

By the method of the stationary phase, after a long but straightforward algebra we obtain

$$\Gamma = \frac{4gC_0}{\omega \cosh^2 k_0 h} \sum_{n=0}^{\infty} \frac{\mathcal{A}_{n0}^R e^{-in\pi/2} \cos n\pi}{H_{nr}^{(1)}(k_0 r) \Big|_{r=R_e}}. \quad (78)$$

This is the Haskind-Hanaoka formula for the OWC in open sea and relates the exciting force Γ with the amplitude of the radiated waves in the direction opposite to the incoming waves $\theta = \pi$. The latter relation is used in the next section to check numerical evaluations for the radiation and diffraction velocity potentials.

3.1. Results and discussion

In this section we examine the effects of the OWC geometry and turbine characteristics on the hydrodynamic behaviour and energy extraction efficiency. For the sake of example, let us consider the following fixed parameters: $A = 1$ m, $h = 10$ m, $\rho = 1000$ kg m⁻³, $\rho_a = 1$ kg m⁻³ and $c_a = 340$ m s⁻¹. Since in the expressions for the velocity potentials (31)-(32)-(54)-(55) there are infinite terms, we need to truncate the summations up to a limiting value $n = N$ and $l = L$ for practical computations. In this work we use $N = L = 100$ to achieve a precision of 2 decimal places (Deng et al., 2013).

3.1.1. Effects of the skirt height

Here we examine the effects of the skirt height h_s on the exciting force Γ , radiation damping \mathcal{B} and added mass \mathcal{C} . Let us fix the following parameters

287 $R_e = h/2$, $\theta_1 = 5\pi/4$ rad, $\theta_2 = 3\pi/4$ rad and $h_c = 0.2 \times h$. The latter
 288 numerical values are the same adopted in Deng et al. (2013), thus we can
 289 perform several validations of our numerical results. As in Deng et al. (2013),
 290 let us define the non-dimensional hydrodynamic quantities

$$\tilde{\Gamma} = \Gamma \frac{\sqrt{g/h}}{gh}, \quad \tilde{\mathcal{B}} = \mathcal{B} \frac{\rho \sqrt{g/h}}{g}, \quad \tilde{\mathcal{C}} = \mathcal{C} \frac{\rho \sqrt{g/h}}{g}, \quad (79)$$

291 and take as a first case $R_i = 0$. Figures 2(a)-2(c) shows the behaviour of $\tilde{\Gamma}$,
 292 $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{C}}$ versus the non-dimensional frequency of the incident waves $\omega^2 h/g$
 293 for five different configurations. Each configuration has a specific value of
 294 the skirt height, that varies from $h - h_c$ to zero. The limiting value in which
 295 $h_s = h - h_c$, corresponds to the case of an OWC supported by a coaxial
 296 tube-sector-shaped structure analysed by Deng et al. (2013), while the case
 297 $h_s = 0$ corresponds to the model developed by Evans and Porter (1997).
 298 Excellent agreement with the numerical results of Deng et al. (2013) (Fig. 4
 299 and Fig. 5, case $d/h = 0.2$) is obtained. This test validates the effectiveness
 300 of the method of solution adopted in this work for the novel device.

301 As in Garrett (1970), Evans and Porter (1997) and Deng et al. (2013),
 302 resonant interactions of the heave and sloshing modes inside the chamber
 303 occur. This is the reason why each peak for each hydrodynamic parameter
 304 is localized at the same frequency. Since $R_i = 0$, the resonances appear ap-
 305 proximately at zeros of the Bessel function $J'_n(k_0 R_e)$ satisfying the boundary
 306 condition for the sloshing modes inside a vertical cylinder of radius R_e and
 307 height h .

308 Note that the OWC without the skirt does not excite the sloshing modes
 309 proportional to $\cos \theta$ or $\sin \theta$. In this case, the first sloshing resonance satisfies
 310 the second zero of $J'_0(k_0 R_e)$ which occurs for $k_0 R_e \simeq 3.831$. Similar results
 311 are obtained in Evans and Porter (1997). On the contrary, if $h_s \neq 0$, we
 312 obtain two additional peaks between the Helmholtz mode and the frequency
 313 corresponding to $k_0 R_e \simeq 3.831$. These peaks are related to the firsts roots of
 314 $J'_1(k_0 R_e)$ and $J'_2(k_0 R_e)$.

315 As the height h_s increases, the first resonant peak of $\tilde{\Gamma}$, $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{C}}$ related
 316 to the Helmholtz mode moves towards small frequencies. In particular, in
 317 the case of $\tilde{\Gamma}$ shown in figure 2(a), the first peak tends to become higher and
 318 sharper as well. On the other hand, the peaks related to the sloshing modes
 319 seem to be unaffected by h_s .

320 The optimal capture factor $C_{F_{opt}}$ (68) is shown in figure 2(d). Within
 321 this range of ω , except for the case without the skirt, four modes are excited

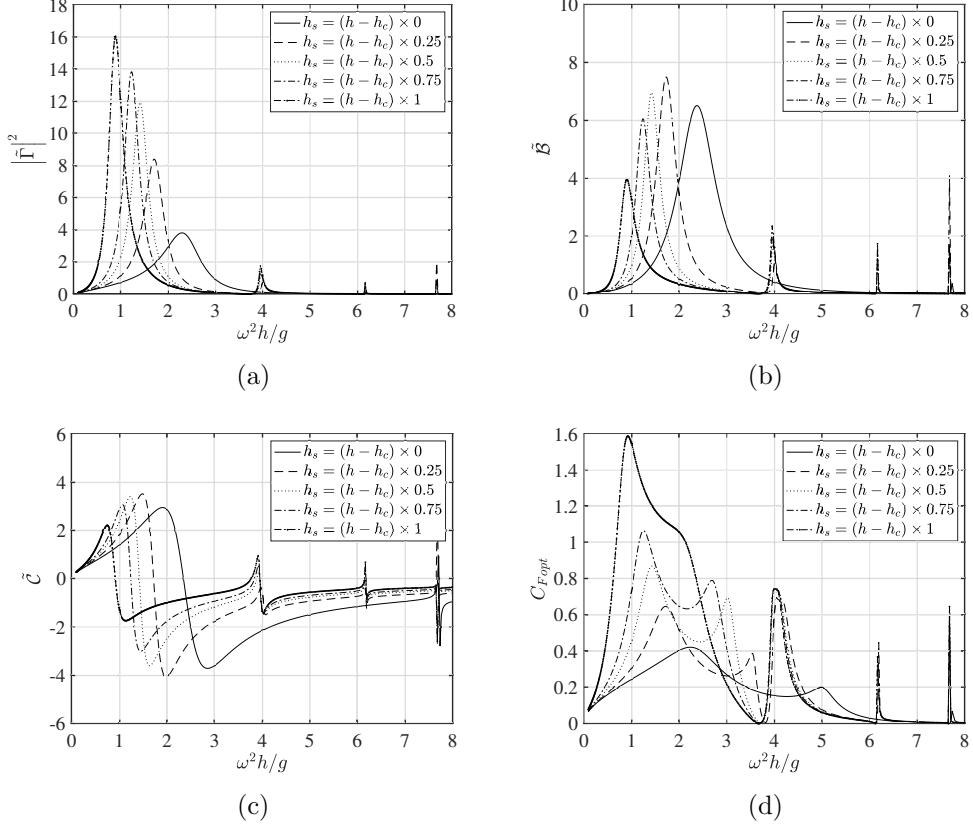


Figure 2: The effects of the skirt height h_s on the hydrodynamic behaviour. 2(a) non-dimensional exciting force $|\tilde{\Gamma}|^2$, 2(b) non-dimensional radiation damping \tilde{B} , 2(c) non-dimensional added mass \tilde{C} and 2(d) optimal capture factor C_{Fopt} of each configuration versus non-dimensional incident wave frequency $\omega^2 h/g$.

hence four maxima occur for C_{Fopt} . The same figure shows that the first resonant peak is the widest and sometimes, depending on h_s , the largest. Values of C_{Fopt} can be larger than 1, i.e. larger than the maximum of a bi-dimensional absorber in a channel flume (Mei et al., 2005). Note that the most efficient configuration corresponds to the case of a skirt extending to the bottom. Next, let the internal radius be $R_i = 0.75 \times R_e$ and evaluate $\tilde{\Gamma}$, $\tilde{\mathcal{B}}$, $\tilde{\mathcal{C}}$ and C_{Fopt} for the same configurations analysed before. Now the internal radius differs from zero, thus the resonant peaks are associated with the sloshing modes of an isolated annular cylinder with fluid occupying the volume Ω_i . Since the general solution of the velocity potential includes both J_n and Y_n the corresponding wave-number k_0 for each sloshing mode must satisfy the following eigenvalue condition

$$J_n(k_0 R_i) Y_n(k_0 R_e) - J_n(k_0 R_e) Y_n(k_0 R_i) = 0. \quad (80)$$

Figure 3 shows that four peaks are present in the computed range of frequencies. Maxima of all the resonant peaks are almost unaffected, while the peaks corresponding to the sloshing modes tend to move towards smaller frequencies. Note also that the second and third peaks for C_{Fopt} increase their width, hence in this case the presence of an internal radius has benefits in terms of power extraction efficiency.

3.1.2. Effects of the skirt opening

Now we analyse the effects of the skirt opening $\theta_2 - \theta_1$ on the same hydrodynamic parameters analysed in the previous section Γ , \mathcal{B} and \mathcal{C} . Let the external radius be $R_e = h/2$ and fix both skirt height $h_s = 0.5 \times (h - h_c)$ and internal radius $R_i = 0$. Five skirt opening angles have been analysed, respectively described by $\theta_1 = \theta_2 = \pi$, $\theta_1 = 5\pi/4$ and $\theta_2 = 3\pi/4$, $\theta_1 = 3\pi/2$ and $\theta_2 = \pi/2$, $\theta_1 = 7\pi/4$ and $\theta_2 = \pi/4$, $\theta_1 = 2\pi$ and $\theta_2 = 0$ rad.

Figure 4 shows the effects of the opening angle for different incident wave non-dimensional frequencies on the hydrodynamic parameters Γ , \mathcal{B} and \mathcal{C} and the optimal capture factor C_{Fopt} . As shown by figures 4(a)-4(c), when the opening increases, the Helmholtz mode resonant peaks decrease while the corresponding resonant frequencies increase. This is less visible for the sloshing modes whose position is almost unvaried. We shall point out that similar results are obtained by Deng et al. (2013) for a skirt extending from the OWC to the sea bottom.

Figure 4(d) shows the behaviour of the optimal capture factor C_{Fopt} . The best configuration with larger and wider peaks corresponds to the symmetric

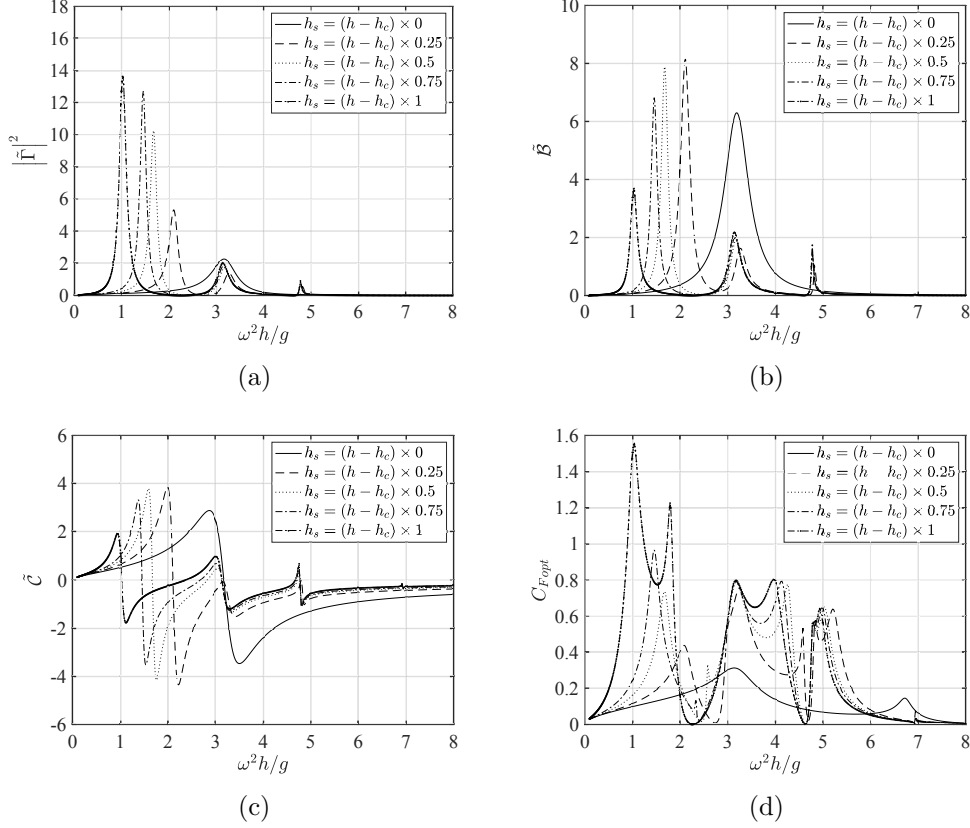


Figure 3: The effects of skirt height h_s and internal radius R_i on the hydrodynamic behaviour. 3(a) non-dimensional exciting force $|\tilde{\Gamma}|^2$, 3(b) non-dimensional radiation damping \tilde{B} , 3(c) non-dimensional added mass \tilde{C} and 3(d) capture factor C_{Fopt} of each configuration versus non-dimensional incident wave frequency $\omega^2 h/g$. The value of the internal radius corresponds to $R_i = 0.75 \times R_e$.

case $\theta_1 = 3\pi/2$ and $\theta_2 = \pi/2$ with opening angle equal to π rad. This result suggests that the skirt plays an important role on the power extraction efficiency, however one should take care of its effects on the OWC structural resistance that could penalise the overall behaviour and durability in real seas. Now we change the internal radius to $R_i = 0.75 \times R_e$. Figures 5(a)-5(d) show $\tilde{\Gamma}$, $\tilde{\mathcal{B}}$, $\tilde{\mathcal{C}}$ and C_{Fopt} versus $\omega^2 h/g$. The same considerations of the previous section can be extended here, i.e. the maximum values of the resonant peaks almost preserve their values, while the peaks of the sloshing modes become wider.

4. Power extraction efficiency in random waves

In this section we investigate the effects of random waves on the generated power. Without loss of generality, we can adopt the JONSWAP spectrum S_ζ to describe the incident wave field (Goda, 2000)

$$S_\zeta(\omega) = \frac{\alpha H_s^2}{\omega} \left(\frac{\omega_p}{\omega}\right)^4 \exp\left[-1, 25 \left(\frac{\omega_p}{\omega}\right)^4\right] \gamma^{\exp[-(\omega/\omega_p-1)^2/(2\sigma)]}, \quad (81)$$

in which H_s is the significant wave height, ω_p denotes the peak frequency and

$$\alpha = \frac{0.0624(1.094 - 0.01915 \ln \gamma)}{0.23 + 0.0336\gamma - 0.185(1, 9 + \gamma)^{-1}}, \quad \sigma = \begin{cases} 0.07 : & \omega \leq \omega_p \\ 0.09 : & \omega > \omega_p \end{cases}, \quad \gamma = 3.3. \quad (82)$$

Because of linearity, the pressure oscillation inside the OWC can be written as

$$P_a(t, \omega_p) = \sum_{n=1}^{\infty} \sqrt{2S_\zeta(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \cos(\omega_n t + \delta_n), \quad (83)$$

where ω_n is the n th component of the discretised spectrum, $\Delta\omega$ is the frequency step, δ_n is a random phase related to ω_n while the term RAO is the response amplitude operator for the air pressure p_a , i.e.

$$\text{RAO}(\omega_n) = \left| \frac{\Gamma(\omega_n)}{\left(\frac{KD}{N\rho_a} + \mathcal{B}(\omega_n)\right) - i\left(\frac{\omega_n V_0}{c_a^2 \rho_a} + \mathcal{C}(\omega_n)\right)} \right|. \quad (84)$$

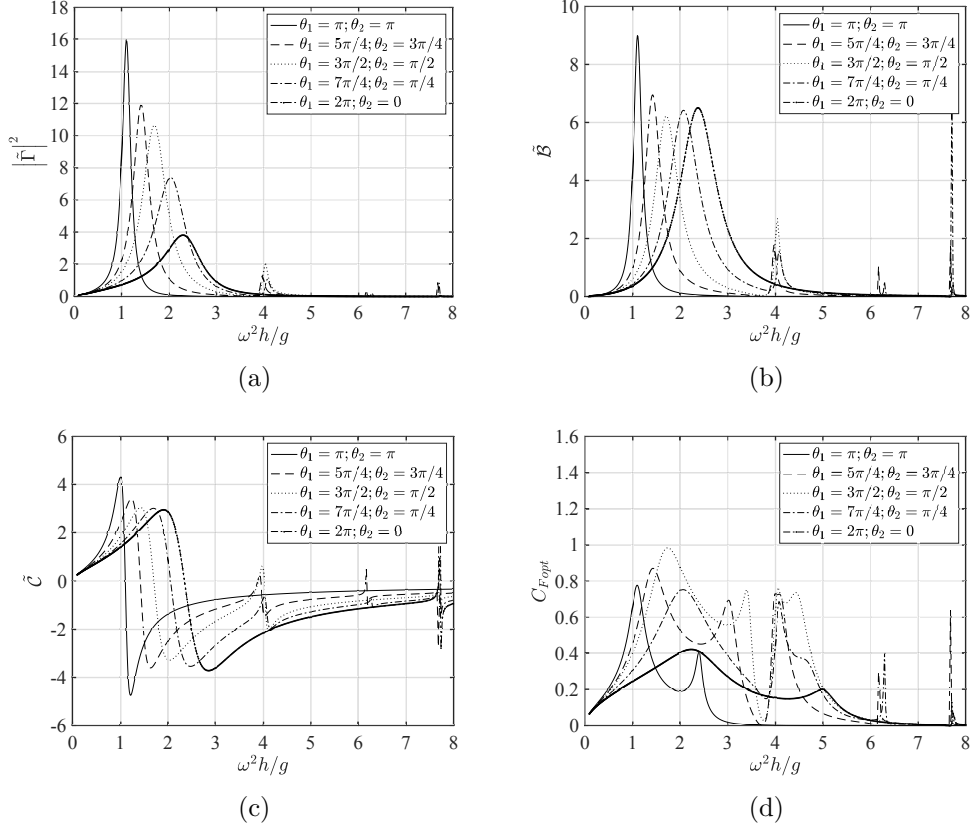


Figure 4: The effects of the skirt opening $\theta_2 - \theta_1$. 4(a) non-dimensional exciting force $|\tilde{\Gamma}|^2$, 4(b) non-dimensional radiation damping \tilde{B} , 4(c) non-dimensional added mass \tilde{C} and 4(d) optimized capture factor C_{Fopt} of each configuration versus non-dimensional incident wave frequency $\omega^2 h/g$.

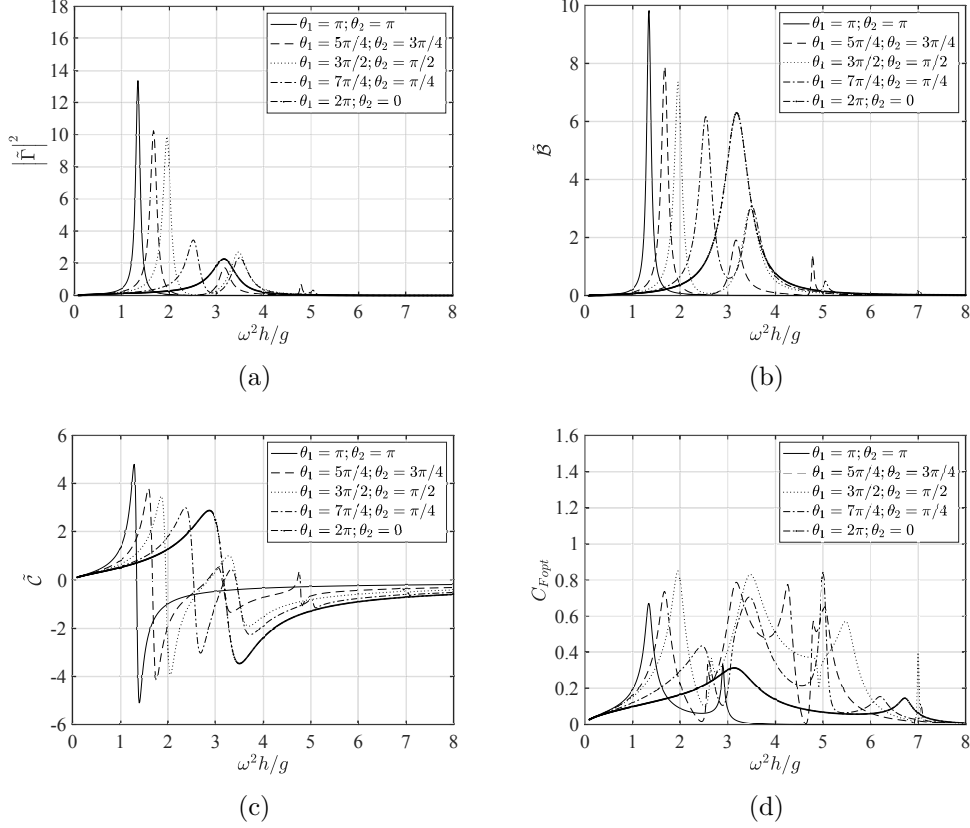


Figure 5: The effects of the skirt opening $\theta_2 - \theta_1$ and internal radius R_i on the hydrodynamic behaviour. 5(a) non-dimensional exciting force $|\tilde{\Gamma}|^2$, 5(b) non-dimensional radiation damping \tilde{B} , 5(c) non-dimensional added mass \tilde{C} and 5(d) optimized capture factor C_{Fopt} of each configuration versus non-dimensional incident wave frequency $\omega^2 h/g$. The value of the internal radius corresponds to $R_i = 0.75 \times R_e$.

377 Then, the instantaneous generated power is

$$\begin{aligned}
P_s(t, \omega_p) = & \frac{KD}{N} \left[\sum_{n=1}^{\infty} \sqrt{2S_{\zeta}(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \cos(\omega_n t + \delta_n) \right]^2 \\
& - \frac{V}{c_a^2} \sum_{n=1}^{\infty} \sqrt{2S_{\zeta}(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \cos(\omega_n t + \delta_n) \\
& \times \sum_{n=1}^{\infty} \sqrt{2S_{\zeta}(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \omega_n \sin(\omega_n t + \delta_n). \quad (85)
\end{aligned}$$

378 From the foregoing expression we obtain the averaged generated power (Michele
379 et al., 2016b)

$$\bar{P}_s(\omega_p) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^{\tau} P_s dt = \frac{KD}{N} \sum_{n=1}^{\infty} S_{\zeta}(\omega_n) \Delta\omega \text{RAO}^2(\omega_n), \quad (86)$$

380 whose expression in the limit $\Delta\omega \rightarrow 0$ becomes

$$\bar{P}_s(\omega_p) = \frac{KD}{N} \int_0^{\infty} S_{\zeta}(\omega) \text{RAO}^2(\omega) d\omega. \quad (87)$$

381 Defining P_{ζ} as the total incident wave power per unit crest width

$$P_{\zeta}(\omega_p) = \int_0^{\infty} \rho g C_g(\omega) S_{\zeta}(\omega) d\omega, \quad (88)$$

382 the capture width ratio in random seas $C_{F\zeta}$ can then be written as

$$C_{F\zeta}(\omega_p) = \frac{\bar{P}_s}{2R_e P_{\zeta}}. \quad (89)$$

383 Let us compare a fixed configuration when excited by random and monochro-
384 matic waves. Here we assume $A = 1$ m, $h = 10$ m, $\rho = 1000$ kg m⁻³, $\rho_a = 1$
385 kg m⁻³, $c_a = 340$ m s⁻¹, the external radius $R_e = h/2$ and two values of
386 the internal radius, $R_i = 0$ and $R_i = 0.75 \times R_e$, respectively. Let us fix
387 the optimal value of $KD/N\rho_a$ that maximizes power extraction for the fixed
388 frequency $\omega = 1$ rad s⁻¹ and assume the symmetric configuration $\theta_1 = 3\pi/2$
389 and $\theta_2 = \pi/2$ rad maximizing power extraction efficiency. In other words, we
390 have fixed both OWC geometry and turbine characteristics and optimized

391 them for a frequency representing the wave climate of a particular area.
 392 This situation can be of practical interest because of the difficulty in tuning
 393 the turbine speed/geometry with a wide range of incident wave frequencies
 394 (López et al., 2014).

395 Figure 6(a) shows the behaviour of $C_{F\zeta}$ and C_F for the case with null
 396 internal radius $R_i = 0$. The abscissa for $C_{F\zeta}$ refers to the peak frequency ω_p
 397 of the JONSWAP spectrum, while the abscissa for C_F refers to the frequency
 398 of the monochromatic incident waves. In both cases the maxima of $C_{F\zeta}$
 399 are smaller than the resonant peaks of C_F , while the system becomes more
 400 efficient outside the resonant frequencies. Furthermore, the narrow peak
 401 related to the resonance of the first sloshing mode decreases significantly
 402 and reduces to a small hump. This fact is consistent with the bad coupling
 403 between the incident wave spectrum and the natural modes characterized
 404 by small radiation damping. Similar results are obtained in the context of
 405 flap-type oscillating wave surge converters by Michele et al. (2016a), Michele
 406 et al. (2016b) and Sarkar et al. (2014).

407 Figure 6(b) shows $C_{F\zeta}$ and C_F respectively versus ω_p and ω for the second
 408 configuration with $R_i = 0.75 \times R_e$. As before, we optimize $KD/N\rho_a$ for the
 409 fixed frequency $\omega = 1 \text{ rad s}^{-1}$. Also in this case the maxima of $C_{F\zeta}$ are
 410 smaller than those of C_F and the spreading effect of the spectrum is evident.
 411 Differently, in the case shown in figure 6(a) the sloshing mode has a significant
 412 contribution because of the small sharpness of the resonant peak in C_F .

413 5. Theoretical and experimental comparisons

414 The damping force exerted by the turbine is usually modelled by an orifice
 415 above the OWC (Perez-Collazo et al., 2018b). In this case, in which an
 416 impulse turbine is used, the relation between the airflow Q through the orifice
 417 and the air pressure P inside the OWC chamber is quadratic (López et al.,
 418 2016), hence the linear relation used to model Wells turbines (62) fails and
 419 cannot be used here. Applying Bernoulli's theorem in correspondence of the
 420 orifice cross section we obtain

$$P_a = \rho_a C_q^2 \frac{Q |Q|}{2\Omega_o}, \quad (90)$$

421 where $C_q \simeq 0.6$ is the dimensionless coefficient of discharge depending on the
 422 orifice geometry and Ω_o is the area of the orifice. Substitution of the latter

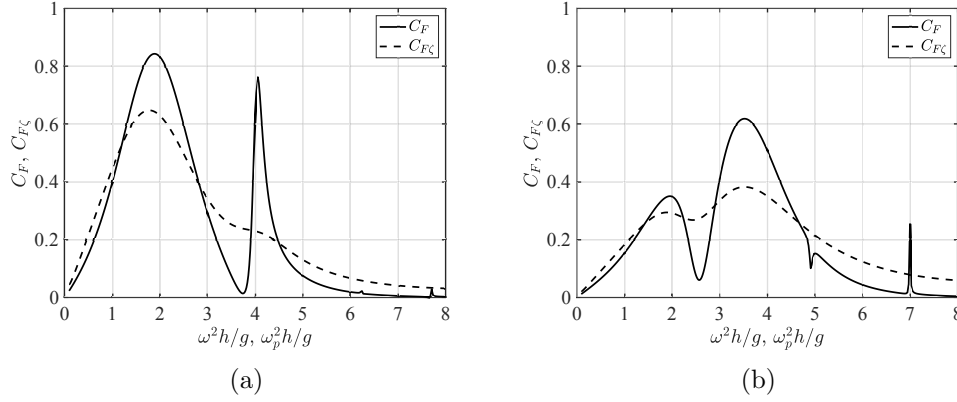


Figure 6: Comparison between the capture factor in random waves $C_{F\zeta}$ and the capture factor for monochromatic incident waves C_F respectively versus peak spectral frequency ω_p and incident frequency ω . Figure 6(a) refers to the configuration with null internal radius R_i while figure 6(b) is related to the case with $R_i = 0.75 \times R_i$. The turbine characteristics are optimized for the frequency $\omega = 1 \text{ rad s}^{-1}$.

423 expression in the nonlinear mixed boundary condition on the free surface
 424 yields (Mei et al., 2005):

$$\Phi_{tt} + g\Phi_z + |\nabla\Phi|_t^2 + \frac{1}{2}\nabla\Phi \cdot \nabla |\nabla\Phi|^2 = -\frac{\rho_a C_q^2}{2\rho\Omega_o^2} \left[\int_{S_i} \Phi_z dS_i \left| \int_{S_i} \Phi_z dS_i \right| \right]_t, \text{ on } S_i. \quad (91)$$

425 Now, by introducing the following non-dimensional quantities denoted by
 426 primes (Michele et al., 2018, 2019a; Michele and Renzi, 2019; Sammarco et
 427 al., 1997a,b):

$$\begin{aligned} (x', y', z') &= (x, y, z) / \lambda, & \Phi' &= \Phi / (A\omega\lambda), & t' &= t\omega, \\ G &= g / (\omega^2\lambda), & \epsilon &= A/\lambda, \end{aligned} \quad (92)$$

428 expression (91) becomes

$$\begin{aligned} \Phi'_{t't'} + G\Phi'_{z'} &= \epsilon |\nabla'\Phi'|_{t'}^2 + \epsilon^2 \frac{1}{2} \nabla'\Phi' \cdot \nabla' |\nabla'\Phi'|^2 \\ &\quad - \epsilon \frac{\rho_a C_q^2}{2\rho\Omega_o^2} \left[\int_{S_i} \Phi'_{z'} dS_i \left| \int_{S_i} \Phi'_{z'} dS_i \right| \right]_{t'}, \text{ on } S_i, \end{aligned} \quad (93)$$

429 thus, if the wave steepness is small, i.e. $\epsilon \ll 1$, and the ratio between the area
 430 S_i and the area of the orifice Ω_o is of order $O(1/\epsilon^2)$, the nonlinear terms on

the right hand side of (93) become small and weak if compared to the linear part on the left hand side. Applying the standard perturbation expansion technique to the velocity potential

$$\Phi' = \Phi'_1 + \epsilon \Phi'_2 + O(\epsilon^2), \quad (94)$$

gives the condition (93) homogeneous and unforced at the leading order $O(1)$:

$$\Phi'_{1_{t't'}} + G\Phi'_{1_{z'}} = 0, \quad \text{on } S_i. \quad (95)$$

If we now return in physical variables and assume both harmonic motion and incident waves at $O(1)$, equation (95) becomes identical to the boundary condition on S_i for ϕ_i^D (26), hence the solution of the velocity potential Φ_1 corresponds to the diffraction velocity potential already found in Section 2.1. As a consequence, the air pressure inside the chamber at the leading order can be approximated by the following expression

$$P_a = \rho_a C_q^2 \frac{\text{Re} \{q^D e^{-i\omega t}\} |\text{Re} \{q^D e^{-i\omega t}\}|}{2\Omega_o^2}. \quad (96)$$

The latter expression yields the averaged rate of work done by the air pressure inside the chamber

$$\overline{P}_{out} = 2\rho_a C_q^2 \frac{|q^D|^3}{3\pi\Omega_o^2}, \quad (97)$$

and the corresponding capture factor

$$C_{Fexp} = \frac{2\rho_a C_q^2 |q^D|^3}{3\pi\Omega_o^2 R_e A^2 \rho g C_g}. \quad (98)$$

In order to validate the theory, comparisons are made with the experimental results of Perez-Collazo et al. (2018b). Channel flume and OWC characteristics are fixed and listed in Table 1.

5.1. Monochromatic waves

Figure 7 shows the values of the capture factor C_{Fexp} versus the wave period T in prototype values for both the analytical (expression (98)) and experimental model (see figure 10 in Perez-Collazo et al. (2018b)). In particular, figure 7(a) and figure 7(b) refer to the different orifice diameters $d_0 = 0.015$ m and $d_0 = 0.019$ m, respectively. The amplitude of the incident

Table 1: Channel and OWC characteristics

Parameters	Symbol	Dimensions
Depth	h	1 m
External radius	R_e	0.08 m
Internal radius	R_i	0 m
OWC draft	h_c	0.076 m
Skirt height	h_s	0.04 m
Skirt angle 1	θ_1	$3\pi/2$ rad
Skirt angle 2	θ_2	$\pi/2$ rad

regular waves is $A = 1$ m. The agreement between both models is good at large periods, however, for the case shown in figure 7(a) the theoretical capture factor is clearly overestimated when $T \in [7, 8]$ s. This is a consequence of the Helmholtz-mode resonance around $T = 5.5$ s. In this range of periods, nonlinearities, viscous dissipation and effects due to vortex shedding at the lower edges (Xu et al., 2016; Xu and Huang, 2019) are not weak anymore and become important. Moreover, the smaller the value of d_0 , the greater the differences between the models. This is because the ratio S_i/Ω_o increases and strengthens the order of magnitude of the last term on the right-hand side of (93).

5.2. Random waves

In this section we analyse the amplitude response of the free surface elevation inside the air chamber in irregular wave conditions. Within the framework of a linearised theory we can write the spectrum of the averaged amplitude response as (Michele et al., 2016a):

$$S_{\bar{\eta}} = \sqrt{2 |\bar{\eta}|^2 S_{\zeta} \Delta\omega}, \quad (99)$$

where $\bar{\eta}$ represents the averaged free-surface amplitude response inside the OWC chamber in monochromatic waves with $A = 1$ m.

For the sake of example, let us consider the configuration with orifice diameter $d_0 = 0.015$ m, significant wave height $H_s = 3.5$ m and peak period $T_p = 13.3$ s in prototype values (Series C07 in Perez-Collazo et al. (2018b)). Figure 8 shows the theoretical and experimental spectra of the averaged amplitude response $S_{\bar{\eta}}$ versus the period T_n of each n th wave component. The

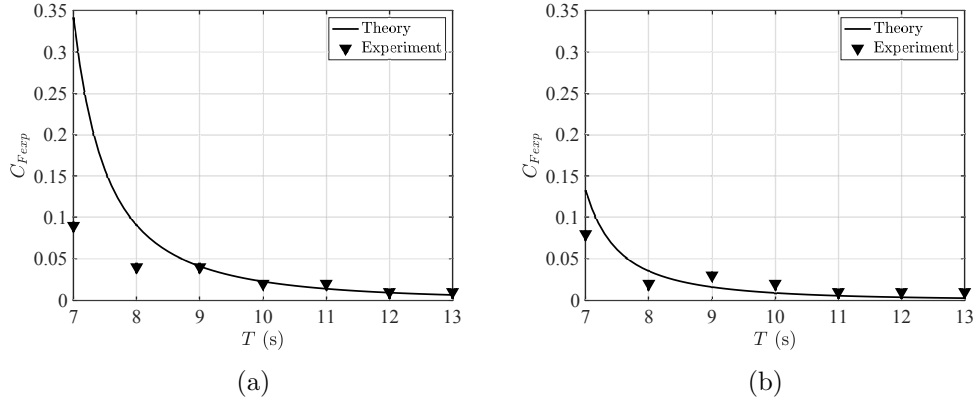


Figure 7: Behaviour of the capture factor C_{Fexp} versus incident wave period T in prototype values for two orifice diameters. 7(a) $d_0 = 0.015$ m, 7(b) $d_0 = 0.019$ m. The solid line indicates the analytical results given by expression (98), while the triangular markers correspond to the experimental results of Perez-Collazo et al. (2018b).

theory predicts one peak around $T_n = 5.5$ s, while the experimental response spectrum tends to decay towards small periods. As in the case of regular waves, this discrepancy is due to the linearised theory that tends to overestimate the amplitude response in resonance conditions. Indeed, the peak is located in correspondence of the Helmholtz pumping mode eigenfrequency. Beyond $T_n = 5.5$ s we stay in the range of validity of the scales (92) and good matching between theory and experiment is obtained.

6. Conclusions

We developed a linearised theory for a cylindrical OWC installed in hybrid wind-wave energy systems. The novel OWC model presented here has a skirt structure integral with the OWC whose task is to increase power extraction efficiency.

We evaluated the dependence of the hydrodynamic quantities such as added inertia, radiation damping and exciting force on the incident wave frequency. Our results show that large resonant peaks occur in correspondence of the frequencies very close to the eigenfrequencies of a cylindrical tank having depth equal to h . Furthermore, we performed a numerical check of the latter quantities and therefore of the accuracy of the results by deriving some useful integral identities based on Green's theorem.

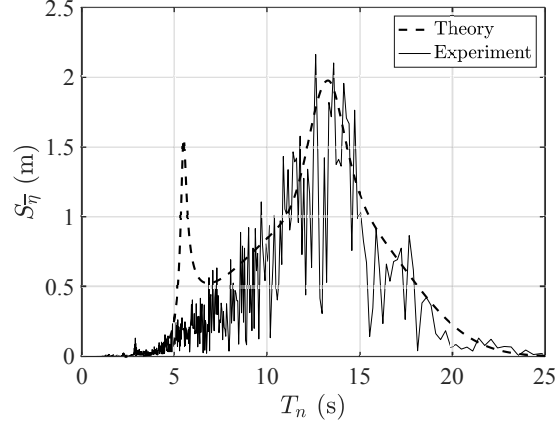


Figure 8: Behaviour of the response amplitude spectrum $S_{\bar{\eta}}$ versus the n th wave component period T_n in prototype values for $H_s = 3.5$ m and peak period $T_p = 13.3$ s. The dashed line represents the analytical results given by (99), while the continuous line corresponds to the spectrum of the time series obtained by Perez-Collazo et al. (2018b).

494 Then we investigated the effects of the skirt height and opening angle on
 495 the hydrodynamic behaviour and efficiency. We found that the greater the
 496 skirt height, the greater the efficiency when the Helmholtz pumping resonates
 497 while the narrow sloshing resonant peaks are almost unaffected and maintain
 498 their shape. This means that the sloshing dynamics depend mainly on the
 499 internal and external OWC radius. Indeed, we showed that when an internal
 500 cylinder is present, wide peaks on the capture factor behaviour can be ob-
 501 tained at large frequencies. Concerning the skirt opening angle, we obtained
 502 that the optimal configuration maximizing power extraction corresponds to
 503 the symmetric case $\theta_1 = 2\pi/3; \theta_2 = \pi/2$ rad.

504 We also investigated the OWC response to random incident waves de-
 505 scribed by the JONSWAP spectrum. We showed that the presence of a
 506 broad range of wave frequencies does not couple well with the narrow res-
 507 onant peaks of some sloshing modes. This is less true for the broad band
 508 Helmholtz-mode at low frequencies. In this case we have large radiation
 509 damping and the resonant peak almost keeps its shape. Outside resonance
 510 the efficiency is larger or comparable to that for the monochromatic case and
 511 the benefits of random waves are evident. Similar results are already well
 512 known for flap-type OWSCs in open sea.

513 Subsequently, we validated the analytical model with the experimental

514 set-up developed by Perez-Collazo et al. (2018b). First, we derived the non-
 515 linear boundary condition on the free surface inside the air chamber. This
 516 condition is completely generalised and therefore valid for any OWC labo-
 517 ratory model that uses orifices to simulate the presence of a turbine. We
 518 solved the problem by applying the perturbation expansion to the velocity
 519 potential and showed that the air pressure and the corresponding airflow
 520 through the orifice depend mainly on the diffraction potential at the leading
 521 order. We evaluated the corresponding theoretical capture factor and com-
 522 pared it with that obtained experimentally by Perez-Collazo et al. (2018b).
 523 Good agreement between both models was found especially for large incident
 524 wave periods and large orifice diameters. Finally, we compared theory and
 525 experiments by analysing the response spectra of the free-surface amplitude
 526 inside the OWC chamber in irregular waves. Good matching was obtained
 527 for frequencies not close to the resonant Helmholtz pumping mode.
 528

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